

Infinitary Formulas in Answer Set Programming

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Abstract

The concept of a stable model for infinitary propositional formulas can be used to define the semantics of answer set programming languages. A semantics of this kind serves as a specification for the latest version of the answer set grounder GRINGO.

The original definition of a stable model [Gelfond and Lifschitz, 1988] has been generalized in several ways (a survey by Lifschitz [2010] provides a comprehensive overview). In particular, it was extended to arbitrary propositional formulas [Pearce, 1997; Ferraris, 2005]. More recently it was extended to propositional formulas with infinite conjunctions and disjunctions [Truszczyński, 2012], and this generalization has been used to define the semantics of aggregates in the input language of the answer set grounder GRINGO¹ [Harrison *et al.*, 2014; Gebser *et al.*, 2015].

Infinitary propositional formulas of rank r , where r is a nonnegative integer, are defined recursively. Propositional atoms are formulas of rank 0. If \mathcal{H} is a set of formulas, and r is the smallest integer that is greater than the ranks of all elements of \mathcal{H} , then both the conjunction over all elements of \mathcal{H} and the disjunction over all elements of \mathcal{H} are formulas of rank r . If r is the smallest integer that is greater than the ranks of F and G then $F \rightarrow G$ is a formula of rank r . The expression $\neg F$ stands for the formula $F \rightarrow \perp$ where \perp is the disjunction over the empty set of formulas and so, is a formula of rank 0. For instance, if Q and $P(\alpha)$, for every α from a non-empty (possibly infinite) set A are atoms then

$$\bigvee_{\alpha \in A} P(\alpha) \tag{1}$$

is a formula of rank 1,

$$\bigvee_{\alpha \in A} P(\alpha) \rightarrow Q \tag{2}$$

is a formula of rank 2, and

$$\bigwedge_{\alpha \in A} \neg P(\alpha) \rightarrow Q \tag{3}$$

¹<http://potassco.sourceforge.net/>

is a formula of rank 3.

The definition of a stable model for a set of infinitary formulas [Truszczyński, 2012] is a straightforward generalization of the definition proposed by Ferraris [2005]. For example, the stable models of (1) are the singletons $\{P(\alpha)\}$ for all α from A .

Rules involving aggregates can be viewed as abbreviations for infinitary propositional formulas. For example, the rule

$$q \leftarrow 1\{p(X)\}$$

(“ q holds if the set p is non-empty”) can be thought of as shorthand for (2), where A is the set of all ground terms. The rule

$$q \leftarrow \{p(X)\}0$$

(“ q holds if the set p is empty”) can be thought of as shorthand for (3). Gebser et al. [2015] use this idea to define the semantics of a large subset of the input language of GRINGO. The translation τ , introduced in that paper, transforms programs in that language into infinitary propositional formulas. This translation is modular: it applies to the program rule by rule. In this sense, it is similar to the process of grounding used in the original definition of a stable model, and differs from the process of “intelligent instantiation”, which is limited to safe programs and is used in the design of answer set solvers.

The stable models of a program Π are then defined as the stable models of the infinitary propositional formula $\tau\Pi$.

The functionality of Version 4.5 of GRINGO, which was released in May of 2015, fully conforms to the semantics defined by Gebser et al. [2015], and this semantics will serve as a specification for future versions. Proofs of correctness of programs with respect to this semantics rely on the theory of stable models of infinitary formulas. In particular, the study of equivalent transformations of programs uses the extension of the theory of strong equivalence [Lifschitz *et al.*, 2001] to infinitary formulas, described by Harrison et al. [2015b]. For example, according to the definition from that paper, the implication (2) is strongly equivalent to the infinite conjunction

$$\bigwedge_{\alpha \in A} (P(\alpha) \rightarrow Q). \quad (4)$$

Strong equivalence of infinitary propositional formulas can be sometimes justified using the concept of an “infinitary instance” of a first-order formula [Harrison *et al.*, 2015a]. For example, (2) is the instance of

$$\exists x P(x) \rightarrow Q$$

if A is the set of object constants of the underlying signature, and (4) is the instance of

$$\forall x(P(x) \rightarrow Q).$$

From the fact that these first-order formulas are intuitionistically equivalent to each other, we can conclude that (2) is indeed strongly equivalent to (4).

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